Physics 211C: Solid State Physics

Instructor: Prof. Tarun Grover   Lecture 15

Topic: Duality and intro to the Toric code, phase diagram of Z_2 gauge theory

1st we convince ourselves that $\exists > 1$ phase of the Z_2 gauge theory.

— Pure Z_2 gauge theory has two phases, despite having no global symmetries.

pure gauge theory:-

$$\mathcal{H} = -h \underbrace{\sum_{\square} \prod_{\square} Z}_{B^2} - J \underbrace{\sum X}_{E^2} \qquad \prod_{+} X = \mathbb{1}$$

Topological order

$\Delta E \sim O(J)$

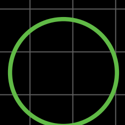
↳ all pnts

J/h

gsd depends on manifold

Top phase transition

"Confined phase"

g.s.d. = 1   gsd = 4 $\equiv$ "Deconfined phase"

$++$ $\longrightarrow$ Energy cost $\approx O(\ell)$ in confined phase

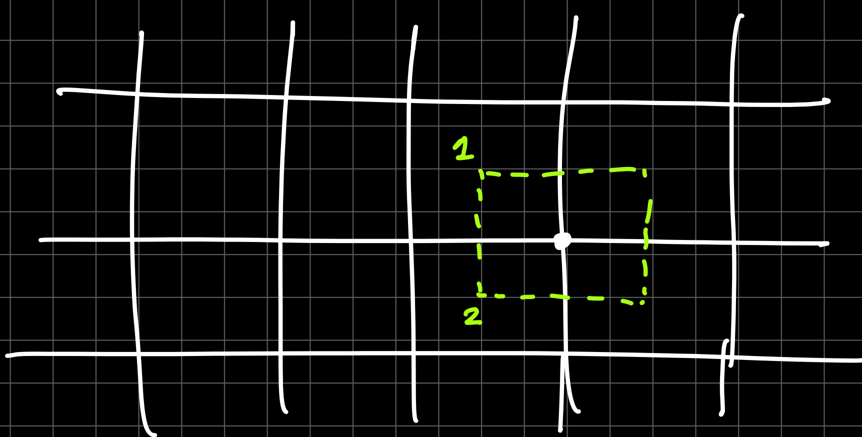
$\longleftarrow\!\!\longrightarrow$

En. cost $\approx O(1)$ in deconfined phase

Duality :-

SE vertices of a $\square$ lattice

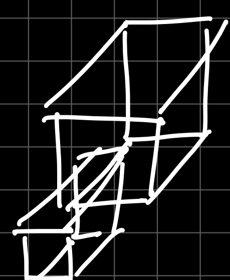
$$\mathcal{H}_{TFIM} = -J\sum_{\langle ij\rangle} s_i^z s_j^z - h\sum s_i^x = -J\sum X - h\sum_{\square}\prod_{\square} Z_{ij}$$



$$\left.\begin{aligned} s_1^z s_2^z &= X_{12} \\ s_i^x &= \prod_{\square\in i} Z \end{aligned}\right\}$$

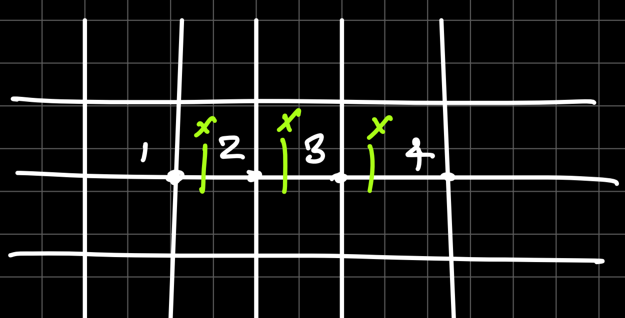
$\exists$ a spacetime version of this too.

$$-\sum s_i^z s_j^z = \sum_{\square}\prod_{\square} Z$$



* Check that Gauss law in the dual gauge theory

Why non-local?

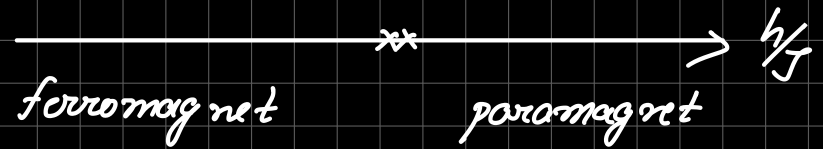


$$S_1^z = S_1^z S_2^z S_2^z S_3^z S_3^z S_4^z S_4^z \cdots$$

$$= X_{12}\, X_{23}\, X_{34} \cdots \quad \text{i.e. a String operator}$$

\# phases of ising model

$$-J \sum s_i^z s_j^z - \sum h\, s^x$$

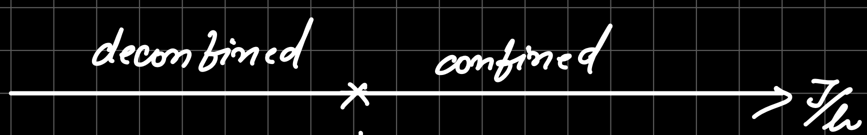


(# of gsd ≠ # of gsd in TFIM)

\# everything is gapped (discrete)

\# critical exponents are same for thermodynamical quantities.

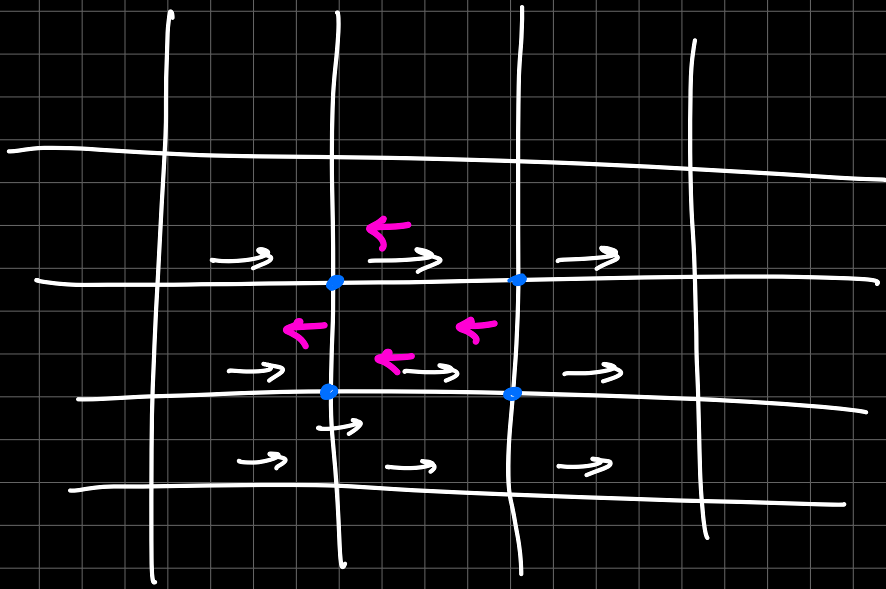
Gauge theory



↳ $\text{Ising}^* = \text{Ising}/\#$

$$-J\, X - h \sum_{\square} Z$$

some discussion



acting with

$\prod_{\square} Z$

makes sense

$\prod_{+} X = 1$

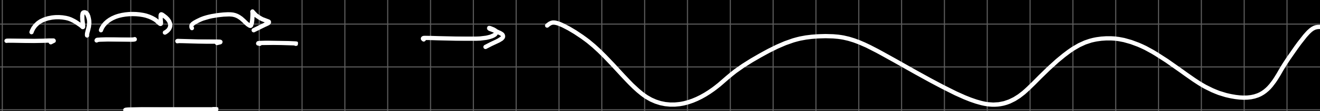
en. cost = 8J

Like spin waves

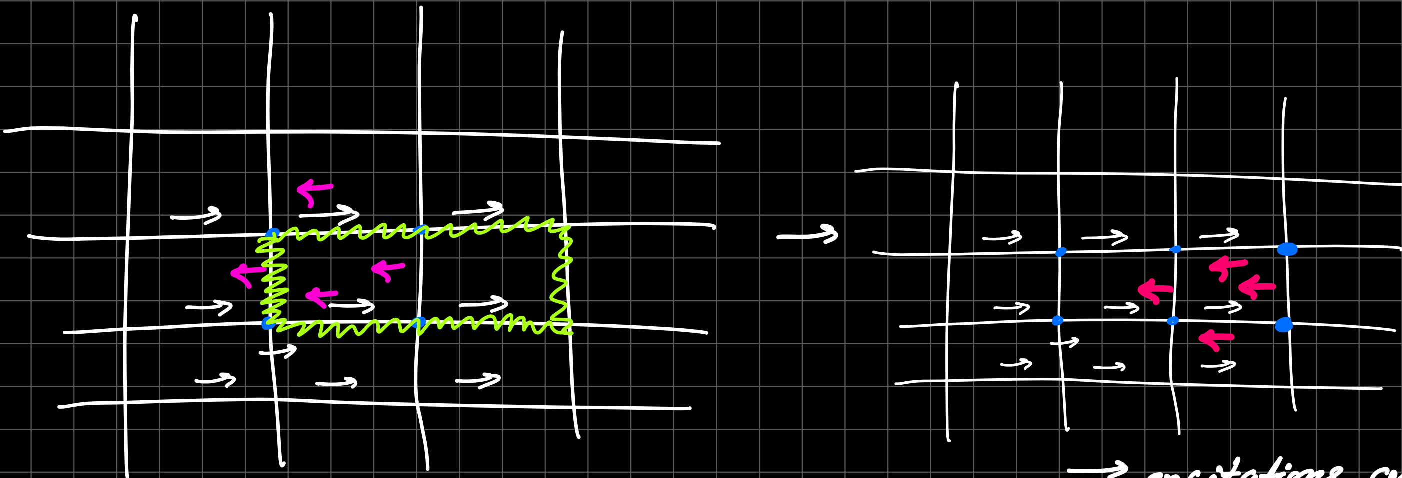
$\uparrow \uparrow \uparrow \_ \_ \_ \_ \uparrow$

$\Downarrow$

$\uparrow \downarrow \uparrow - - - - \uparrow \iff \uparrow\uparrow - - - \downarrow \uparrow\uparrow \quad - - -$

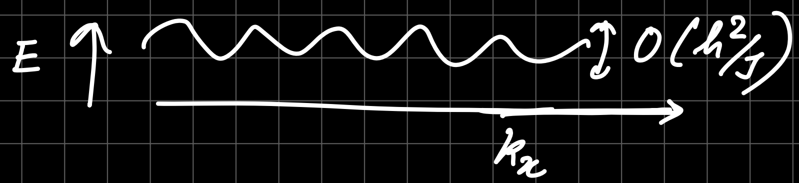


degenerate states $\longrightarrow$ band.



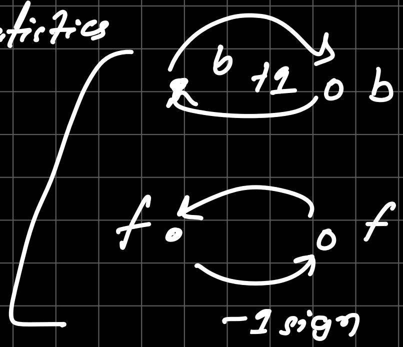
$\longrightarrow$ excitations can just move around.

$$H_{eff} = -\frac{h^2}{J} \sum_{\substack{\square \\ 2\times1}} \prod_{\substack{\square \\ 2\times1}} Z - \frac{h^2}{J} \sum_{\substack{\square \\ 1\times2}} \prod_{\substack{\square \\ 1\times2}} Z$$

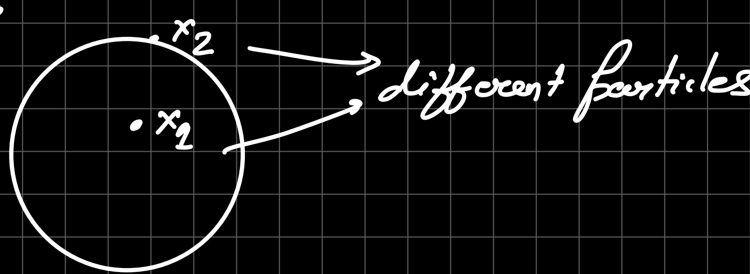


* What about Statistics?

exchange statistics



Braiding



t_{ij} = hopping op. that moves a QP from i to j.

[Guessing statistics from simple arguments]

* In large J (confined phase), q.p.'s are bosonic.

$h \gg J$

$J = 0$

$$\mathcal{H} = -h \sum_{\square} \prod_{\square} Z \qquad \text{with} \qquad \prod_{+} X = 1$$

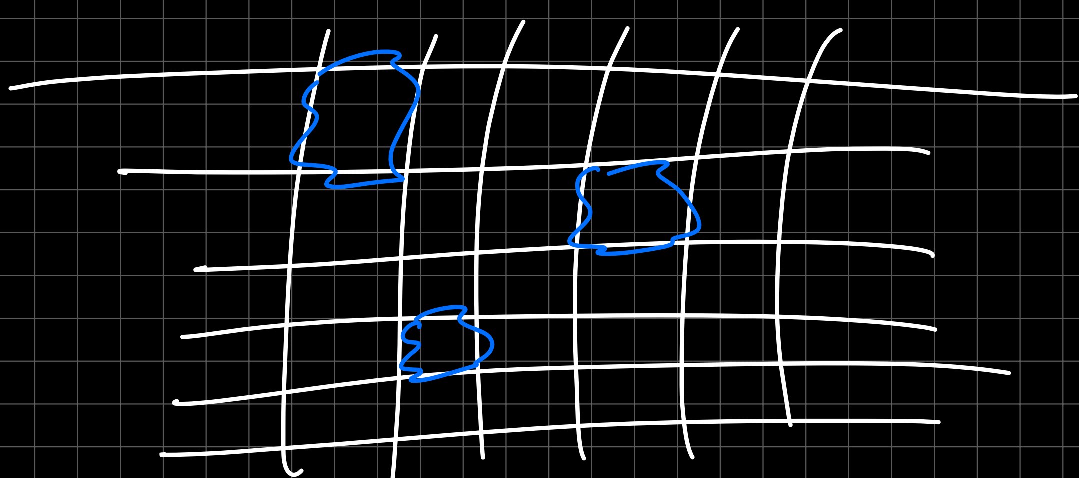
Add a constraint to the hamiltonian

$$\mathcal{H} = -\underbrace{\sum_{\square} \prod_{\square} Z}_{B^2} - \underbrace{\sum_{+} \prod_{+} X}_{(\nabla \cdot E)^2}$$

$$\frac{1 + \prod_{\square} Z}{2} \qquad \frac{1 + \prod_{+} X}{2}$$

sum of constant projectors.

"Kitaev's toric code"



$= |\psi\rangle$

non-local operators diff.

&

Stability against perturbations.